# Effects of the quantity $\sigma_{TS}$ on the spin structure functions of nucleons in the resonance region

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**Abstract**

In this paper, we investigate the effects of the quantity $\sigma_{TS}$ on the spin-structure functions of nucleons in the resonance region. The Schwinger sum rule for the spin structure function $g_2(x,Q^2)$ at the real photon limit is derived for the nucleon treated as a composite system, and it provides a crucial constraint on the longitudinal transition operator which has not been treated consistently in the literature. The longitudinal amplitude $S_{\frac{1}{2}}$ is evaluated in the quark model with the transition operator that generates the Schwinger sum rule. The numerical results of the quantity $\sigma_{TS}$ are presented for both spin structure functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$ in the resonance region. Our results show that this quantity plays an important role in the low $Q^2$ region, which can be tested in the future experiments at CEBAF.

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# 1. Introduction

The quantity $\sigma_{TS}$, defined in the spin structure functions of nucleons

$$g_1(x,Q^2) = \frac{M_T K}{8\pi^2\alpha(1+\frac{Q^2}{\omega^2})}\left[\sigma_{1/2}(\omega,Q^2) - \sigma_{3/2}(\omega,Q^2) + \frac{2\sqrt{Q^2}}{\omega}\sigma_{TS}(\omega,Q^2)\right] \tag{1}$$

and

$$g_2(x,Q^2) = \frac{M_T K}{8\pi^2\alpha(1+\frac{Q^2}{\omega^2})}\left[\frac{2\omega}{\sqrt{Q^2}}\sigma_{TS}(\omega,Q^2) - (\sigma_{1/2}(\omega,Q^2) - \sigma_{3/2}(\omega,Q^2))\right], \tag{2}$$

where K is the photon flux, $x$ the scaling variable, and $M_T$ the nucleon mass, was not fully investigated due to the fact that most studies were concentrated in the deep inelastic scattering region, where the quantity $\frac{2\sqrt{Q^2}}{\omega}\sigma_{TS}(\omega,Q^2)$ in $g_1(x,Q^2)$ vanishes. This is nolonger the case recently as there have been growing interests in studying the spin structure functions in the small $Q^2$ region, where the resonance contributions are important. Consequently, the investigation of the effects of the quantity $\sigma_{TS}$ in the small $Q^2$ region has become increasingly important.

Such a program began with the suggestion[1] that the $Q^2$ dependence of the spin dependent sum rule might play a significant role in the interpretation of the European Muon Collaboration(EMC) data[2], which starts with a negative Drell-Hearn-Gerasimov(DHG)[3] sum rule in the real photon limit and ends with a positive sum rule[4] in the large $Q^2$limit[5]:

$$\int_0^1 g_1(x,Q^2)dx = \begin{cases} -\frac{\omega_{th}}{4M_T}\kappa^2 & Q^2=0 \\ \Gamma & Q^2\to\infty \end{cases} \tag{3}$$

where

$$\omega_{th} = \frac{Q^2+2m_\pi M+m_\pi^2}{2M} \tag{4}$$

is the threshold energy of pion photoproductions, $\kappa$ the anomalous magnetic moment, and $\Gamma$ a positive quantity. Because the contributions from the quantity $\sigma_{TS}$ to the sum rule in Eq. 3 also vanish in the real photon limit, most quantitative studies[5, 6, 7] of the $Q^2$ dependence of the sum rule in Eq. 3 were concentrated on the contributions from the quantity $\sigma_{\frac{1}{2}} - \sigma_{\frac{3}{2}}$ in $g_{1,2}(x,Q^2)$. Indeed, these investigations have shown a strong $Q^2$ dependence of the sum rule in the $Q^2 \leq 2.5$ GeV$^2$ region. However, the study by Soffer and Teryaev[8] suggested that the quantity $\sigma_{TS}$ plays a significant role in the small $Q^2$ region,

which is highlighted by another set of sum rules for the spin structure function $g_2(x,Q^2)$;

$$\int_0^1 g_2(x,Q^2)dx = \begin{cases} \frac{\omega_{th}}{4M_T}\kappa(\kappa+e_T) & Q^2=0 \\ 0 & Q^2\to\infty, \end{cases} \tag{5}$$

in which the same kinematics in Eq. 3 is used. The sum rules in Eq. 5 were first derived by Schwinger[9] in the real photon limit and Burkhardt and Cummingham[10] in the large $Q^2$ limit. Combining Eqs. 3 and 5 leads to the sum rule for the quantity $\sigma_{TS}$ in the real photon limit;

$$\lim_{Q^2\to 0}\int_{\omega_{th}}^{\infty}\sigma_{TS}\frac{d\omega}{\sqrt{Q^2}} = \frac{4\pi^2\alpha}{4M_T^2}e_T\kappa. \tag{6}$$

The magnitude of the sum rule for the quantity $\sigma_{TS}$ in the real photon limit is certainly comparable to the DHG sum rule. Thus, a more quantitative study of the contributions from the quantity $\sigma_{TS}$ to the spin structure functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$ is called for. Such a study not only enables us to give a more precise estimate of the $Q^2$ dependence of the sum rule for $g_1(x,Q^2)$, but also provides a quantitative calculation of the sum rule for $g_2(x,Q^2)$ for the first time in the quark model. The focus of this paper is to develop a framework in the quark model to evaluate the contributions from the quantity $\sigma_{TS}$, and present the numerical results that can be tested in the future CEBAF experiments.

The sum rules in Eqs. 3 and 5 are more general and model independent, therefore, they must be satisfied in the quark model in order to give a consistent evaluation of the spin structure functions in the resonance region. This has been investigated[6] in the quark model for sum rules in Eq. 3, and they require that the electromagnetic interaction for a many body system should has the form

$$\begin{aligned} H_t &= \sum_j\{e_j\vec{r}_j\cdot\vec{E}_j - \frac{e_j}{2m_j}\vec{\sigma}_j\cdot\vec{B}_j - \frac{e_j}{4m_j}\vec{\sigma}_j\cdot[\vec{E}_j\times\frac{\vec{p}_j}{2m_j} - \frac{\vec{p}_j}{2m_j}\times\vec{E}_j] \\ &\quad + \sum_{j<l}\frac{1}{4M_T}[\frac{\vec{\sigma}_j}{m_j} - \frac{\vec{\sigma}_l}{m_l}]\cdot(e_l\vec{E}_l\times\vec{p}_j - e_j\vec{E}_j\times\vec{p}_l)\}, \end{aligned} \tag{7}$$

and the quantity $\Gamma$ in Eq. 3 is related to the quark model matrix element

$$\Gamma = \frac{1}{2}\langle i|\sum_j e_j^2\sigma_j^z|i\rangle_{P-A} \tag{8}$$

where quark $j$ at position $r_j$ has mass $m_j$ and charge $e_j$, and $A(P)$ indicates that the directions of the polarization between photons and the target are antiparallel(parallel). On the other hand, the sum rule for the quantity $\sigma_{TS}$ in

Eq. 6 has not been investigated in the quark model. The derivation of Eq. 6 in the quark model is by no means trivial since it was proved[9] in QED by assuming the nucleon as an elementary particle. The similar transition of the DHG sum rule from an elementary particle to a many body system led to extensive discussions in late sixties and early seventies[11]. Moreover, the proof of Eq. 6 requires evaluations of both helicity amplitude $A_{\frac{1}{2}}$ and the longitudinal amplitude $S_{\frac{1}{2}}$. While the helicity amplitude $A_{\frac{1}{2}}$ has been calculated[12] with the transition operator $H_t$ in Eq. 7, the longitudinal amplitude $S_{\frac{1}{2}}$ has not been treated consistently in the literature. In particular, the problem of the current conservation was not fully understood[13], and an *ad hoc* current $J_3' = -\frac{k_3 J_3 - k_0 J_0}{k_3}$ was introduced[14] to evaluate the longitudinal transitions of the baryon resonances[15]. The sum rule for the quantity $\sigma_{TS}$ provides an important test to the quark model; the consistency requires that the sum rules for both $g_1(x, Q^2)$ and $g_2(x, Q^2)$ be generated by the same set of transition operators for a many body system. In section 2, we show that Eq. 6 is indeed generated by the electromagnetic interaction $H_t$ in Eq. 7, which also satisfies the DHG sum rule. The longitudinal transition operator is obtained by requiring it satisfying the sum rule in Eq. 6, which is not only gauge invariant, but also consistent with $H_t$ in Eq. 7. In particular, the spin-orbit interaction and the Wigner rotation that are crucial to the DHG sum rule for a many body system should be present in the longitudinal transition operator.

In section 3, we show that the quantity $\sigma_{TS}$ in $g_2(x, Q^2)$ cancels the transverse cross section $\sigma_{\frac{1}{2}} - \sigma_{\frac{3}{2}}$ in the large $Q^2$ limit, which leads to the well known Burkhardt-Cottingham sum rule[10] for the spin structure function $g_2(x, Q^2)$. Thus, a consistent framework to evaluate the quantity $\sigma_{TS}$ is established. In section 4, we evaluate the longitudinal amplitude $S_{\frac{1}{2}}$ with the transition operator that generates the sum rule for the quantity $\sigma_{TS}$, which has not been done systematically in the literature. The numerical results for the spin structure functions $g_1(x, Q^2)$ and $g_2(x, Q^2)$ in the small $Q^2$ region are also shown in section 4. Our results show that the effects of the quantity $\sigma_{TS}$ on the spin structure functions are important in the resonance region. Finally, the conclusion is given in Section 5.

# 2. The Sum Rule For The Quantity $\sigma_{TS}$

Because the spin structure functions of the nucleon are usually measured above the pion photoproduction threshold, the sum rule for the quantity $\sigma_{TS}$ can be

formulated as[5]

$$\int_0^1 (g_1(x, Q^2 = 0) + g_2(x, Q^2 = 0))dx = \lim_{Q^2 \to 0} \frac{M\omega_{th}}{4\pi^2\alpha} \int_{\omega_{th}}^{\infty} \sigma_{TS} \frac{d\omega}{\sqrt{Q^2}}. \qquad (9)$$

The cross section $\sigma_{TS}$ in Eq. 9 can be expressed in terms of the transverse and longitudinal helicity amplitudes, which is

$$\begin{aligned}\sigma_{TS} = \frac{\pi}{\sqrt{2}} \sum_{f>i} \{ & \langle i, \frac{1}{2} \mid H_l^* \mid f\rangle\langle f \mid H_t \mid i, -\frac{1}{2}\rangle \\ & + \langle i, -\frac{1}{2} \mid H_t^* \mid f\rangle\langle f, \mid H_l \mid i, \frac{1}{2}\rangle\}\delta(\omega - \omega_f), \end{aligned} \qquad (10)$$

where $H_t$ is the transverse transition operator, and the longitudinal transition operator $H_l$ is defined as

$$H_l = \epsilon_0 J_0 - \epsilon_3 J_3. \qquad (11)$$

Using the gauge invariant condition, $k_\mu J^\mu = k_\mu \epsilon^\mu = 0$, and choosing the longitudinal polarization vector $\epsilon_\mu$ as

$$\epsilon_\mu^L = \{\epsilon_0, 0, 0, \epsilon_3\} = \left\{ \frac{k_3}{\sqrt{Q^2}}, 0, 0, \frac{\omega}{\sqrt{Q^2}} \right\}, \qquad (12)$$

we have

$$\langle f \mid H_l \mid i\rangle = \frac{\sqrt{Q^2}}{\omega} \langle f \mid J_3 \mid i\rangle, \qquad (13)$$

or

$$\langle f|H_l|i\rangle = \frac{\sqrt{Q^2}}{k} \langle f|J_0|i\rangle. \qquad (14)$$

Both Eqs. 13 and 14 can be used in the evaluation of the quantity $\sigma_{TS}$ because of the current conservation. It can be shown that the quantity $\sigma_{TS}$ is independent of which longitudinal current being used. Consequently, the sum rule for the quantity $\sigma_{TS}$ does not depend on the choice of the longitudinal current as well, as long as it is gauge invariant.

Substitute Eq. 10 into Eq. 9, we have

$$\begin{aligned}\int_{\omega_{th}}^{\infty} \sigma_{TS} \frac{d\omega}{\sqrt{Q^2}} = \frac{\pi}{\sqrt{2}\omega} \sum_{f>i} \{ & \langle i, \frac{1}{2} \mid J_3^* \mid f\rangle\langle f \mid H_t \mid i, -\frac{1}{2}\rangle \\ & + \langle i, -\frac{1}{2} \mid H_t^* \mid f\rangle\langle f \mid J_3 \mid i, \frac{1}{2}\rangle\}. \end{aligned} \qquad (15)$$

The operator $H_t$ in Eq. 15 is also responsible to generate the DHG[3] sum rule for the transverse cross section $\sigma_{\frac{1}{2}} - \sigma_{\frac{3}{2}}$ in the real photon limit. Thus, the

consistency requires that the same $H_t$ in Eq. 7 should be also used in deriving Eq. 6. Following the same procedure as that in Ref. [6], we rewrite Eq. 7 by separating the center of mass motion from the internal motion:

$$H_t = \sqrt{\frac{\omega}{2}}(h^c + h^p), \tag{16}$$

where

$$h^c = i\sum_j \left\{ e_j \vec{R}\cdot\vec{\epsilon} - \frac{1}{4M_T}(\frac{2e_j}{m_j} - \frac{e_T}{M_T})\vec{\sigma}_j\cdot(\vec{\epsilon}\times\vec{P}_T)\right\} + \hat{\mu}^c, \tag{17}$$

and

$$h^p = i\sum_j \left\{ e_j\vec{\epsilon}\cdot(\vec{r}_j - \vec{R}) - \frac{1}{4}(\frac{e_j}{m_j} - \frac{e_T}{M_T})\vec{\sigma}_j\cdot(\vec{\epsilon}\times(\frac{\vec{p}_j}{m_j} - \frac{\vec{P}_T}{M_T}))\right\} + \hat{\mu}^p, \tag{18}$$

where $\epsilon = -\frac{1}{\sqrt{2}}(1, i, 0)$ is the transverse polarized vector of photons. The last terms $\hat{\mu}^c$ and $\hat{\mu}^p$ in Eqs. 17 and 18 correspond to the second term in Eq. 7. Their contributions to $\sigma_{TS}$ is more complicated, and will be evaluated separately.

The longitudinal transition operator $J_3$ can be obtained by simply replacing the polarization vector $\vec{\epsilon}$ with the vector $\hat{\vec{k}}$, and the separation of the center of mass from the internal motions for the longitudinal transition is therefore easy to follow

$$J_3 = \sqrt{\frac{\omega}{2}}(j^c + j^p), \tag{19}$$

where

$$j^c = \sum_j \left\{ ie_j\vec{R}\cdot\hat{\vec{k}} + \frac{i}{4M_T}(\frac{e_T}{M_T} - \frac{2e_j}{m_j})\vec{\sigma}_j\cdot(\hat{\vec{k}}\times\vec{P}_T)\right\}, \tag{20}$$

and

$$j^p = \sum_j \left\{ i\hat{\vec{k}}\cdot(\vec{r}_j - \vec{R})e_j + \frac{i}{4}(\frac{e_T}{M_T} - \frac{e_j}{m_j})\vec{\sigma}_j\cdot(\hat{\vec{k}}\times(\frac{\vec{p}_j}{m_j} - \frac{\vec{P}_T}{M_T}))\right\}. \tag{21}$$

Now we are in the position to derive Eq. 6 from the constituent quark model. Substituting Eqs. 16 and 19 into Eq. 15, we have

$$\begin{aligned}\int_{\omega_{th}}^{\infty}\sigma_{TS}\frac{d\omega}{\sqrt{Q^2}} = \frac{4\pi^2\alpha}{2\sqrt{2}}\sum_{f>i}\{&\langle i,1/2 \mid j^{c*} \mid f\rangle\langle f \mid h^c \mid i,-1/2\rangle \\ &+\langle i,-1/2 \mid h^{c*} \mid f\rangle\langle f \mid j^c \mid i,1/2\rangle \\ &+\langle i,1/2 \mid j^{p*} \mid f\rangle\langle f \mid h^p \mid i,-1/2\rangle \\ &+\langle i,-1/2 \mid h^{p*} \mid f\rangle\langle f \mid j^p \mid i,1/2\rangle\},\end{aligned} \tag{22}$$

where the charge $e^2$ has been written as $4\pi\alpha$ explicitly so that the total charge for protons becomes 1 instead of $e$. Using the closure relation, Eq. 22 becomes

$$\sum_{f>i}\{\langle i,\frac{1}{2}\mid j^{c*}\mid f\rangle\langle f\mid h^c\mid i,-\frac{1}{2}\rangle+\langle i,-\frac{1}{2}\mid h^{c*}\mid f\rangle\langle f\mid j^c\mid i,\frac{1}{2}\rangle$$
$$+\langle i,\frac{1}{2}\mid j^{p*}\mid f\rangle\langle f\mid h^p\mid i,-\frac{1}{2}\rangle+\langle i,-\frac{1}{2}\mid h^{p*}\mid f\rangle\langle f\mid j^p\mid i,\frac{1}{2}\rangle\}$$
$$=\langle i,\frac{1}{2}\mid j^{c*}h^c+j^{p*}h^p\mid i,-\frac{1}{2}\rangle+\langle i,-\frac{1}{2}\mid h^{c*}j^c+h^{p*}j^p\mid i,\frac{1}{2}\rangle$$
$$-\langle i,\frac{1}{2}\mid j^{c*}\mid i\rangle\langle i\mid h^c\mid i,-\frac{1}{2}\rangle-\langle i,-\frac{1}{2}\mid h^{c*}\mid i\rangle\langle i,\mid j^c\mid i,\frac{1}{2}\rangle. \tag{23}$$

We first consider the correlations between the longitudinal transition operators, $j^c(j^p)$, and the first term $h^c-\mu^c(h^p-\mu^p)$ in Eq. 17 (18). In the Appendix we show that

$$\langle i,\frac{1}{2}\mid j^{c*}(h^c-\hat{\mu}^c)\mid i-\frac{1}{2}\rangle+\langle i,-\frac{1}{2}\mid(h^{c*}-\hat{\mu^c}^*)j^c\mid i,\frac{1}{2}\rangle$$
$$=-\langle i,\frac{1}{2}\mid\sum_j\sigma_j^+\frac{e_T}{2M_T}\left(\frac{2e_j}{m_j}-\frac{e_T}{M_T}\right)\mid i,-\frac{1}{2}\rangle\sqrt{2}, \tag{24}$$

and similarly

$$\langle i,\frac{1}{2}\mid j^{p*}(h^p-\hat{\mu}^p)\mid i,-\frac{1}{2}\rangle+\langle i,-1/2\mid(h^{p*}-\hat{\mu^p}^*)j^p\mid i,\frac{1}{2}\rangle$$
$$=\langle i,\frac{1}{2}\mid\sum_j\sigma_j^+\left(\frac{e_j}{m_j}-\frac{e_T}{M_T}\right)\left(\frac{e_T}{2M_T}-\frac{e_j}{2m_j}\right)\mid i,-\frac{1}{2}\rangle\sqrt{2}. \tag{25}$$

We turn to the correlation between the magnetic term $\hat{\mu}$ and the longitudinal transition operator $j$. The leading term for the operator $\mu$ is

$$\mu_0^c=\sum_j\frac{e_j}{2m_j}\vec{\sigma}_j\cdot(\vec{\epsilon}\times\hat{\vec{k}}), \tag{26}$$

and

$$\mu_0^p=0. \tag{27}$$

The correlation between $\mu_0^c$ and $j^c$ gives

$$\langle i,\frac{1}{2}\mid j^{c*}\hat{\mu}_0^c\mid i,-\frac{1}{2}\rangle+\langle i,-\frac{1}{2}\mid\hat{\mu}_0^{c*}j^c\mid i,\frac{1}{2}\rangle$$
$$=\sum_j\Big\{\langle i,\frac{1}{2}\mid e_T\vec{R}\cdot\hat{\vec{k}}\frac{e_j}{2m_j}\vec{\sigma}_j\cdot(\vec{\epsilon}\times\hat{\vec{k}})\mid i,-\frac{1}{2}\rangle$$
$$+\langle i,-\frac{1}{2}\mid\frac{e_j}{2m_j}\vec{\sigma}_j\cdot(\vec{\epsilon}^*\times\hat{\vec{k}})e_T\vec{R}\cdot\hat{\vec{k}}\mid i\frac{1}{2}\rangle\Big\}, \tag{28}$$

Substitute the polarization $\epsilon = -\frac{1}{\sqrt{2}}(1, i, 0)$ and $\epsilon^* = -\frac{1}{\sqrt{2}}(1, -i, 0)$ into Eq. 28, we find that the two terms in Eq. 28 cancel each other so that it vanishes. Thus, the nonzero contribution from the correlation between $\hat{\mu}$ and the longitudinal transition $j$ should come from the next order. By expanding the photon wave function $e^{i\vec{k}\cdot\vec{r}_j}$, we have

$$\hat{\mu} = \sum_j \frac{e_j}{2m_j}\vec{\sigma}_j \cdot (\vec{\epsilon} \times \hat{\vec{k}})(1 + i\vec{k} \cdot \vec{r}_j + O(k)). \tag{29}$$

Because the leading term in Eq. 29 gives a zero contribution to the quantity $\sigma_{TS}$, we examine the second term in Eq. 29. In the real photon limit, we rewrite the second term in Eq. 29 as

$$\begin{aligned}\langle f \mid i \sum_j \frac{e_j}{2m_j}\vec{\sigma}_j \cdot (\vec{\epsilon} \times \hat{\vec{k}})\vec{k} \cdot \vec{r}_j \mid i\rangle &= i\langle f \mid [H, \sum_j \frac{e_j}{2m_j}\vec{\sigma}_j \cdot (\vec{\epsilon} \times \hat{\vec{k}})\hat{\vec{k}} \cdot \vec{r}_j] \mid i\rangle \\ &= \langle f \mid \sum_j \frac{e_j}{2m_j}\vec{\sigma}_j \cdot (\vec{\epsilon} \times \hat{\vec{k}})\hat{\vec{k}} \cdot \frac{\vec{p}_j}{m_j} \mid i\rangle,\end{aligned} \tag{30}$$

so that the closure relation could be used, because the transition operator has no explicit dependence on the transition energy $\omega$. The operator $H$ in Eq. 30 is the Hamiltonian of the system;

$$H = \sum_j \frac{\vec{p}_j^2}{2m_j} + \sum_{i,j} V_{ij}(\vec{r}_i - \vec{r}_j). \tag{31}$$

Therefore, $\hat{\mu}^c$ and $\hat{\mu}^p$ in Eqs. 17 and 18 are

$$\hat{\mu}_1^c = \sum_j \frac{e_j}{2m_j}\vec{\sigma}_j \cdot (\vec{\epsilon} \times \hat{\vec{k}})\hat{\vec{k}} \cdot \frac{\vec{P}_T}{M_T} \tag{32}$$

and

$$\hat{\mu}_1^p = \sum_j \frac{e_j}{2m_j}\vec{\sigma}_j \cdot (\vec{\epsilon} \times \hat{\vec{k}})\hat{\vec{k}} \cdot (\frac{\vec{p}_j}{m_j} - \frac{\vec{P}_T}{M_T}). \tag{33}$$

The correlation between $\hat{\mu}_1^{c,p}$ and $j^{c,p}$ gives

$$\begin{aligned}&\langle i, \frac{1}{2} \mid j^{c*}\hat{\mu}_1^c + j^{p*}\hat{\mu}_1^p \mid i, -\frac{1}{2}\rangle + \langle i, -\frac{1}{2} \mid \hat{\mu^c}^*_1 j^c + \hat{\mu^p}^*_1 j^p \mid i, \frac{1}{2}\rangle \\ &= \langle i, \frac{1}{2} \mid \sum_j \sigma_j^+ \left[\frac{e_T}{2M_T}\frac{e_j}{m_j} + \left(\frac{e_j}{m_j} - \frac{e_T}{M_T}\right)\frac{e_j}{2m_j}\right] \mid i, -\frac{1}{2}\rangle\sqrt{2}.\end{aligned} \tag{34}$$

This shows that the nonzero contributions to the correlation between the magnetic transition $\hat{\mu}$ and the longitudinal transition operator $j$ come from

the higher order expansion in $\hat{\mu}$, this feature does not exist in the transverse transverse correlations that leads to the DHG sum rule.

Therefore, combine Eqs. 25, 28 and 34, we have

$$\langle i, \frac{1}{2} \mid j^{c*}h^c + j^{p*}h^p \mid i, -\frac{1}{2}\rangle + \langle i, -\frac{1}{2} \mid h^{c*}j^c + h^{p*}j^p \mid i, \frac{1}{2}\rangle = 0. \tag{35}$$

That is, the sum rule for the quantity $\sigma_{TS}$ is only determined by the static properties of ground states:

$$\int_{\omega_{th}}^{\infty} \sigma_{TS} \frac{d\omega}{\sqrt{Q^2}} = -\frac{4\pi^2\alpha}{2\sqrt{2}} \Big\{ \langle i, \frac{1}{2} \mid j^{c*} \mid i\rangle\langle i \mid h^c \mid i, -\frac{1}{2}\rangle \tag{36}$$
$$+\langle i, -\frac{1}{2} \mid h^{c*} \mid i\rangle\langle i \mid j^c \mid i, \frac{1}{2}\rangle \Big\}.$$

This is a general feature for the sum rules of both $g_1(x, Q^2)$ and $g_2(x, Q^2)$; the sum rules in real photon limit do not depend on the internal structure of the nucleon so that it behaves like an elementary particle in the low energy limit. Using the relation

$$\langle i | \sum_j \frac{e_j}{2m_j} \vec{\sigma}_j | i\rangle = \mu \langle i | \vec{\sigma}_T | i\rangle \tag{37}$$

where $\vec{\sigma}_T$ is the total spin operator of a many-body system, we have

$$\langle i | h^c | i\rangle = \langle i | H^c | i\rangle \tag{38}$$

and

$$\langle i | j^c | i\rangle = \langle i | J^c | i\rangle, \tag{39}$$

where

$$H^c = i\Big\{ e_T \vec{R}\cdot\vec{\epsilon} + \mu\vec{\sigma}_T\cdot(\vec{\epsilon}\times\hat{\vec{k}})\frac{\vec{P}_T\cdot\hat{\vec{k}}}{M_T} - \frac{1}{2M_T}\left(2\mu - \frac{e_T}{2M_T}\right)\vec{\sigma}_T\cdot(\vec{\epsilon}\times\vec{P}_T)\Big\} \tag{40}$$

and

$$J^c = i\Big\{ e_T \vec{R}\cdot\vec{\epsilon} - \frac{1}{2M_T}\left(2\mu - \frac{e_T}{2M_T}\right)\vec{\sigma}_T\cdot(\vec{\epsilon}\times\vec{P}_T)\Big\}. \tag{41}$$

Thus, the closure relation can be used because the operators $H^c$ and $J^c$ do not connect the ground state with the excited states:

$$\langle i, \frac{1}{2} | j^{c*} | i\rangle\langle i | h^c | i, -\frac{1}{2}\rangle + \langle i, -\frac{1}{2} | h^{c*} | i\rangle\langle i | j^c | i, \frac{1}{2}\rangle$$
$$= \langle i, \frac{1}{2} | J^{c*}H^c | i, -\frac{1}{2}\rangle + \langle i, -\frac{1}{2} | H^{c*}J^c | i, \frac{1}{2}\rangle$$
$$= \frac{\sqrt{2}}{2M_T^2} e_T \kappa, \tag{42}$$

which leads to the sum rule in Eq. 6. Consequently, the sum rule for the spin structure function $g_2$ in the real photon limit is just a linear combination of Eq. 6 and the DHG sum rule:

$$\lim_{Q^2\to 0}\int_0^1 dx g_2(x,Q^2) = \frac{\omega_{th}}{M_T}\frac{\kappa(\kappa+e_T)}{4}. \tag{43}$$

This shows that the sum rules for both $g_1(x,Q^2)$ and $g_2(x,Q^2)$ in the real photon limit can be derived consistently from the same set of the transition operators in the quark model. It also highlights the importance of the spin-orbit interaction and the nonadditive term in both transverse and longitudinal transition operators $H_t$ and $J_3$. In the next section, we will give an intuitive proof that the same is also true for the sum rules in the large $Q^2$ limit.

# 3. The Extension To The Large $Q^2$ limit

In the case of $Q^2 \neq 0$, an extra term is generated from the transverse operator $h = h^c + h^p$ so that

$$h = h_0 + h_1, \tag{44}$$

where $h_0$ represents the transition operator $h$ at $Q^2 = 0$, and

$$h_1 = \sum_j \frac{Q^2}{\omega^2}\frac{e_j}{2m_j}\vec{\sigma}_j\cdot(\vec{\epsilon}\times\hat{\vec{k}})\hat{\vec{k}}\cdot\frac{\vec{p}_j}{m_j}, \tag{45}$$

while the longitudinal operator $j = j^c + j^p$ remains the same. Eq. 35 shows that the correlation between $h_0$ and $j$ is zero for the inclusive processes, thus only the correlation between $h_1$ and $j$ needs to be investigated. Note that the Bjorken scaling variable $x_j$ is related to the photon energy and the mass of partons[6]:

$$x_j = \frac{Q^2}{2M_T\omega} = \frac{m_j}{M_T} \tag{46}$$

in the large $Q^2$ limit. The operator $h_1$ can be written as

$$h_1 = \sum_j \frac{2}{Q^2} e_j\vec{\sigma}_j\cdot(\vec{\epsilon}\times\hat{\vec{k}})\hat{\vec{k}}\cdot\vec{p}_j. \tag{47}$$

The correlation between $h_1$ and $j$ gives

$$\begin{aligned}&\langle i,\frac{1}{2}\mid j^* h_1\mid i,-\frac{1}{2}\rangle + \langle i,-\frac{1}{2}\mid h_1^* j\mid i,\frac{1}{2}\rangle\\ &\qquad = \frac{2\sqrt{2}}{Q^2}\langle i,\frac{1}{2}|\sum_j e_j^2\sigma_j^+|i,-\frac{1}{2}\rangle. \end{aligned}\tag{48}$$

Therefore, we have

$$\lim_{Q^2\to\infty}\int_{\omega_{th}}^{\infty}\sigma_{TS}\frac{d\omega}{\sqrt{Q^2}}=\frac{4\pi^2\alpha}{Q^2}\langle i,\frac{1}{2}|\sum_j e_j^2\sigma_j^+|i,-\frac{1}{2}\rangle. \tag{49}$$

A similar procedure in the large $Q^2$ extension of the DHG sum rule gives

$$\int_{\omega_{th}}^{\infty}(\sigma_{\frac{1}{2}}-\sigma_{\frac{3}{2}})\frac{d\omega}{\omega}=\frac{4\pi^2\alpha}{Q^2}\langle i|\sum_j e_j^2\sigma_j^z|i\rangle_{P-A}. \tag{50}$$

Combining Eqs. 49 and 50 gives the well known Burkhardt-Cottingham(BC) sum rule[10] for the spin structure function $g_2$,

$$\int_0^1 g_2(x)dx=0. \tag{51}$$

Therefore, the sum rules for both $g_1(x,Q^2)$ and $g_2(x,Q^2)$ can be obtained from the same set of electromagnetic interactions in Eqs. 7 and 19 for a many body system. It shows that the transition of the spin dependent sum rules (both DHG sum rule and Eq. 6) from the real photon limit to the large $Q^2$ limit is an evolution from an exclusive, coherent elastic scattering to an inclusive, incoherent deep-inelastic scattering of a many-body system. Moreover, by reproducing the spin-dependent sum rules in the real photon and large $Q^2$ limits, we are able to establish a framework to evaluate the spin structure functions of nucleons in the finite $Q^2$ region, where the contributions from baryon resonances are important, and the quark model has been very successful.

# 4. The evaluation of the spin dependent sum rules in the low $Q^2$ region

The numerical studies of the quantity $\sigma_{TS}$ require the evaluations of the transverse helicity amplitude $A_{\frac{1}{2}}$ and the longitudinal amplitude $S_{\frac{1}{2}}$. The helicity amplitude $A_{\frac{1}{2}}$ has been calculated[12] by using the transition operator in Eq. 7 that generates the DHG sum rule. Thus, only the longitudinal amplitude $S_{\frac{1}{2}}$ needs to be evaluated. Following Eq. 14, the longitudinal transition amplitude $S_{\frac{1}{2}}$ is

$$S_{\frac{1}{2}}=\langle f|J_0|i\rangle, \tag{52}$$

and we have the longitudinal transition operator[16]

$$
\begin{aligned}
J_0 &= \sqrt{\frac{1}{2\omega}}\{\sum_j (e_j + \frac{ie_j}{4m_j^2}\vec{k}\cdot(\vec{\sigma}_j\times\vec{p}_j))e^{i\vec{k}\cdot\vec{r}_j}) \\
&\quad -\sum_{j<l}\frac{i}{4M_T}(\frac{\vec{\sigma}_j}{m_j}-\frac{\vec{\sigma}_l}{m_l})\cdot(e_j\vec{k}\times\vec{p}_l e^{i\vec{k}\cdot\vec{r}_j} - e_l\vec{k}\times\vec{p}_j e^{i\vec{k}\cdot\vec{r}_l})\}, \qquad (53)
\end{aligned}
$$

where the second and third terms are the spin-orbit and nonadditive terms that represents the relativistic corrections to the leading charge operator. The study in previous sections clearly shows that the spin-orbit and the nonadditive terms are crucial in reproducing the sum rule for the quantity $\sigma_{TS}$.

Because the longitudinal amplitude $S_{\frac{1}{2}}$ of baryon resonances has not been systematically calculated with the transition operator $J_0$ in Eq. 53, we show the analytical expressions of the longitudinal amplitudes $S_{\frac{1}{2}}$ between the nucleon and baryon resonances in $SU(6)\otimes O(3)$ symmetry limit in Table 1. The evaluation of the $Q^2$ dependence of the longitudinal amplitudes $S_{\frac{1}{2}}$ follows the procedure of Foster and Hughes[17], and the the longitudinal amplitudes $S_{\frac{1}{2}}$ as a function of $Q^2$ for the resonance $S_{11}(1535)$, $D_{13}(1520)$ and $F_{15}(1688)$ are shown in Fig. 1. These results are in better agreement with the analysis by Gerhardt[18] than the previous calculations[15], who extracted the longitudinal amplitudes from the electroproduction data. The numerical results in Fig. 1 shows that the longitudinal amplitudes are quite large in the low $Q^2$ region, thus suggest that they play a significantly role in the spin structure functions of nucleon in the low $Q^2$ region.

The resonance contributions to the sum rules of the spin structure functions can be expressed in terms of the helicity amplitudes, $A_{\frac{1}{2}}$ and $A_{\frac{3}{2}}$, and the longitudinal amplitudes $S_{\frac{1}{2}}$:

$$
\int g_1(x,Q^2)dx = \sum_R K.E.\left[|A^R_{\frac{1}{2}}|^2 - |A^R_{\frac{3}{2}}|^2 + \frac{Q^2}{\sqrt{2}\omega k}(S^{R^*}_{\frac{1}{2}}A^R_{\frac{1}{2}} + A^{R^*}_{\frac{1}{2}}S^R_{\frac{1}{2}})\right] \qquad (54)
$$

and

$$
\int g_2(x,Q^2)dx = \sum_R K.E.\left[\frac{\omega}{\sqrt{2}k}(S^{R^*}_{\frac{1}{2}}A^R_{\frac{1}{2}} + A^{R^*}_{\frac{1}{2}}S^R_{\frac{1}{2}}) - \left(|A^R_{\frac{1}{2}}|^2 - |A^R_{\frac{3}{2}}|^2\right)\right], \qquad (55)
$$

where the kinetic factor K.E. is

$$
K.E. = \frac{M\omega_{th}}{4\pi\alpha(1+\frac{Q^2}{\omega^2})\omega} \qquad (56)
$$

and $\omega_{th}$ is given in Eq. 4. The total width of each resonance is treated as zero so that the integration over the photon energy can be approximated by a summation over all the resonances. The background contributions from the nucleon born terms in the single pion photoproductions are not included in Eqs. 54 and Eq. 55, and they can be easily included later in a more detail studies. Because these amplitudes are evaluated by using the transition operators that generate the the spin dependent sum rules, the calculations of the spin structure function in the resonance region become straightforward. In Fig. 2, we show the resonance contributions to the sum rule for $g_1(x, Q^2)$, in which every resonance below 2 GeV is included. The resonances $P_{11}(1440)$ and $P_{33}(1600)$ are treated as hybrid states[19], and the study shows[20] that the $Q^2$ dependence of the transition amplitudes for hybrid $P_{11}(1440)$ and $P_{33}(1600)$ gives a better agreement with the existing data. The numerical result for the $Q^2$ dependence of the integral for the transverse cross section $\sigma_{\frac{1}{2}} - \sigma_{\frac{3}{2}}$ is also shown in Fig. 2, and it is in good agreement with a more sophisticated evaluation in Ref. [5]. The resonance contribution to the integral $\int g_1(x, Q^2 = 0)dx$ at the real photon limit is -0.121, which is in good agreement with the theoretical prediction $-\frac{\omega_{th}}{4M_T}\kappa^2$ with $k_p = 1.79$ for the proton target[5]. This result is consistent with the conclusions of our previous investigation[6]; the contributions from resonances, in particular the resonance $P_{33}(1232)$, dominate the DHG sum rule. The difference between the $g_1(x, Q^2)$ sum rule and the contribution from the quantity $\sigma_{\frac{1}{2}} - \sigma_{\frac{3}{2}}$ shows the importance of the quantity $\sigma_{TS}$. It is particularly significant in the small $Q^2$ region, and the addition of the quantity $\sigma_{TS}$ has pushed the crossing point that the sum rule is zero from 0.7 GeV$^2$ to around 0.5 GeV$^2$.

The sum rule for the spin structure function $g_2(x, Q^2)$ in the resonance region is shown in Fig. 3. The resonance contributions to the sum rule $\int g_2(x, Q^2)$ at the real photon limit is 0.182, while Eq.43 gives 0.192 for the proton target. This shows that the resonance contributions dominate the sum rule for $g_2(x, Q^2)$ in the real photon limit as well. The $g_2(x, Q^2)$ is only significant in the $Q^2 \leq 1$ GeV$^2$ region, and decreases very quickly as $Q^2$ increases. There is also a sign change for the sum rule of $g_2(x, Q^2)$ at $Q^2 \approx 1$ GeV$^2$. A recent calculation[21] in the single pion channel of pion photoproduction has shown a similar behaviour, in which only the nucleon born term is considered. This behaviour is not consistent with the $Q^2$ dependence of the sum rule of $g_2(x, Q^2)$ derived in Ref. [9, 8]. It may represent the theoretical uncertainty of the quark model calculations. On the other hand, it would be very interesting to see if there is a sign change in the experimental data.

To highlight the importance of the quantity $\sigma_{TS}$ in the resonance region,

we present an estimate of the total sum rule for $g_1(x, Q^2)$ by including the contributions from outside the resonance region. Following the procedure in Ref. [5], the total spin dependent sum rule should be written as

$$\int_0^1 g_1(x, Q^2) = \int_{x_r}^1 + \int_0^{x_r} g_1(x, Q^2) dx, \tag{57}$$

where

$$x_r = \frac{Q^2 + 2m_\pi M_T + m_\pi^2}{W_r^2 + Q^2 - M_T^2} \tag{58}$$

with $W_r = 2.0$ GeV. The first term in Eq. 57 represents the contributions from the resonance region, it shows that the contributions from the resonance region does not cover the whole kinetic region from $x = 0$ to $x = 1$. The second term in Eq. 57 comes from the outside resonance region, and we showed in Ref. [5] that this term becomes increasingly important as $Q^2$ increases. Because there is no experimental information on the quantity $\sigma_{TS}$ outside the resonance region, one could only make a qualitative estimate on the second term in Eq. 57. We show the estimate of the $Q^2$ dependence of the spin dependent sum rule $\int_0^1 g_1(x, Q^2) dx$ in Fig. 4. The contribution from the second term is obtained from the estimate of the nonresonant contribution in Ref. [5], in which the quantity $\sigma_{TS}$ is not included. Thus, this estimate could only be regarded as a lower limit of the spin dependent sum rule for $g_1(x, Q^2)$. Nevertheless, the effects of the quantity $\sigma_{TS}$ on the $Q^2$ dependence of the sum rule $\int_0^1 g_1(x, Q^2) dx$ are very important, and it could not be neglected if the high twist term that generates the leading $1/Q^2$ corrections to the spin structure function in the deep inelastic scattering region is extracted from $Q^2 \approx 1.5 \sim 2.5$ GeV$^2$ region.

# 5. Conclusion

We have presented a consistent framework to investigate the spin structure functions of nucleon in the resonance region, which the model independent sum rules in the real photo limit and the large $Q^2$ limit are satisfied. We show that the same set of transition operator generates both DHG sum rule for the transverse cross section, $\sigma_{\frac{1}{2}} - \sigma_{\frac{3}{2}}$, and the sum rule for the quantity $\sigma_{TS}$. The sum rule for the quantity $\sigma_{TS}$ also provides a crucial constraint on the longitudinal transition operator; it requires the longitudinal transition operator to be gauge invariant and to be expanded to order $O(\frac{v^2}{c^2})$ consistently. The operator in Eq. 53 satisfies these requirements. This clarifies some of the problems in the literature on the longitudinal transitions, although the problem of the model space truncation discussed in Ref. [13] is not considered here.

A more quantitative calculation of the spin dependent sum rules for both spin structure functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$ are presented for the first time in the quark model. Our numerical results indicate that the effects of the quantity $\sigma_{TS}$ are very important in small $Q^2$ region, which certainly can be tested in the future experiments at CEBAF[22].


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# Appendix

The terms that contribute to the spin flip in Eq. 24 are

$$\begin{aligned}
&\langle i,\frac{1}{2}|j^{c*}(h^c-\hat{\mu^c})|i,-\frac{1}{2}\rangle+\langle i,-\frac{1}{2}|(h^c-\hat{\mu^c})^*j^c|i,\frac{1}{2}\rangle= \\
&\sum_j\langle i,\frac{1}{2}|\frac{e_T}{4M_T}\left(\frac{e_T}{M_T}-\frac{2e_j}{m_j}\right)\vec{\sigma}_j\cdot(\hat{\vec{k}}\times\vec{P}_T)\vec{R}\cdot\vec{\epsilon}|i,-\frac{1}{2}\rangle \\
&+\sum_j\langle i,-\frac{1}{2}|\frac{e_T}{4M_T}\left(\frac{e_T}{M_T}-\frac{2e_j}{m_j}\right)\vec{R}\cdot\vec{\epsilon}^*\vec{\sigma}_j\cdot(\hat{\vec{k}}\times\vec{P}_T)|i,\frac{1}{2}\rangle \\
&+\sum_j\langle i,\frac{1}{2}|\frac{e_T}{4M_T}\left(\frac{e_T}{M_T}-\frac{2e_j}{m_j}\right)\vec{R}\cdot\hat{\vec{k}}\vec{\sigma}_j\cdot(\vec{\epsilon}\times\vec{P}_T)|i,-\frac{1}{2}\rangle \\
&+\sum_j\langle i,-\frac{1}{2}|\frac{e_T}{4M_T}\left(\frac{e_T}{M_T}-\frac{2e_j}{m_j}\right)\vec{\sigma}_j\cdot(\vec{\epsilon}^*\times\vec{P}_T)\vec{R}\cdot\hat{\vec{k}}|i,\frac{1}{2}\rangle
\end{aligned} \tag{59}$$

Lets consider the terms proportional to $\vec{R}\cdot\vec{\epsilon}$ in Eq. 59. The product $\vec{\sigma}_j\cdot(\hat{\vec{k}}\times\vec{P}_T)$ can be written as

$$\vec{\sigma}_j\cdot(\hat{\vec{k}}\times\vec{P}_T)=-i\sigma_j^+(P_x-iP_y)+i\sigma_j^-(P_x+iP_y), \tag{60}$$

where $\sigma^\pm=\frac{1}{2}(\sigma_x\pm i\sigma_y)$. Substitute $\epsilon=-\frac{1}{\sqrt{2}}(1,i,0)$ into Eq. 59, we have

$$\begin{aligned}
&\sum_j\langle i,\frac{1}{2}|\frac{e_T}{4M_T}\left(\frac{e_T}{M_T}-\frac{2e_j}{m_j}\right)\vec{\sigma}_j\cdot(\hat{\vec{k}}\times\vec{P}_T)\vec{R}\cdot\vec{\epsilon}|i,-\frac{1}{2}\rangle \\
&+\sum_j\langle i,-\frac{1}{2}|\frac{e_T}{4M_T}\left(\frac{e_T}{M_T}-\frac{2e_j}{m_j}\right)\vec{R}\cdot\vec{\epsilon}^*\vec{\sigma}_j\cdot(\hat{\vec{k}}\times\vec{P}_T)|i,\frac{1}{2}\rangle
\end{aligned}$$

$$= \sum_j \langle i, \frac{1}{2} | \frac{e_T}{4M_T} \left( \frac{e_T}{M_T} - \frac{2e_j}{m_j} \right) \sigma_j^+ \frac{i}{\sqrt{2}} P^- R^+ | i, -\frac{1}{2} \rangle$$

$$- \sum_j \langle i, -\frac{1}{2} | \frac{e_T}{4M_T} \left( \frac{e_T}{M_T} - \frac{2e_j}{m_j} \right) \sigma_j^- \frac{i}{\sqrt{2}} R^- P^+ | i, \frac{1}{2} \rangle, \tag{61}$$

where $P^\pm = P_x \pm iP_y$ and $R^\pm = R_x \pm iR_y$. Notice that for the total 1/2 initial and final states

$$\langle i, \frac{1}{2} | \sigma_j^+ | i, -\frac{1}{2} \rangle = \langle i, -\frac{1}{2} | \sigma_j^- | i, \frac{1}{2} \rangle \tag{62}$$

in our convention for the Pauli matrix $\sigma^\pm$ and the spin wave functions. Eq. 61 becomes

$$\sum_j \langle i, \frac{1}{2} | \frac{e_T}{4M_T} \left( \frac{e_T}{M_T} - \frac{2e_j}{m_j} \right) \sigma_j^+ \frac{i}{\sqrt{2}} \left[ P^- R^+ - R^- P^+ \right] | i, -\frac{1}{2} \rangle$$

$$= \sum_j \langle i, \frac{1}{2} | \frac{e_T}{4M_T} \left( \frac{e_T}{M_T} - \frac{2e_j}{m_j} \right) \sigma_j^+ \sqrt{2} \left[ 1 + iR_y P_x - iR_x P_y \right] | i, -\frac{1}{2} \rangle \tag{63}$$

where the term $R_y P_x - R_x P_y$ is an angular momentum operator for the center of mass motions of nucleons, which is zero in this process. Thus, we have

$$\sum_j \langle i, \frac{1}{2} | \frac{e_T}{4M_T} \left( \frac{e_T}{M_T} - \frac{2e_j}{m_j} \right) \vec{\sigma}_j \cdot (\hat{\vec{k}} \times \vec{P}_T) \vec{R} \cdot \vec{\epsilon} | i, -\frac{1}{2} \rangle$$

$$+ \sum_j \langle i, -\frac{1}{2} | \frac{e_T}{4M_T} \left( \frac{e_T}{M_T} - \frac{2e_j}{m_j} \right) \vec{R} \cdot \vec{\epsilon}^* \vec{\sigma}_j \cdot (\hat{\vec{k}} \times \vec{P}_T) | i, \frac{1}{2} \rangle$$

$$= \sum_j \langle i, \frac{1}{2} | \frac{e_T}{4M_T} \left( \frac{e_T}{M_T} - \frac{2e_j}{m_j} \right) \sigma_j^+ \sqrt{2} | i, -\frac{1}{2} \rangle. \tag{64}$$

Taking the same procedure, we have

$$\sum_j \langle i, \frac{1}{2} | \frac{e_T}{4M_T} \left( \frac{e_T}{M_T} - \frac{2e_j}{m_j} \right) \vec{R} \cdot \hat{\vec{k}} \vec{\sigma}_j \cdot (\vec{\epsilon} \times \vec{P}_T) | i, -\frac{1}{2} \rangle$$

$$+ \sum_j \langle i, -\frac{1}{2} | \frac{e_T}{4M_T} \left( \frac{e_T}{M_T} - \frac{2e_j}{m_j} \right) \vec{\sigma}_j \cdot (\vec{\epsilon}^* \times \vec{P}_T) \vec{R} \cdot \hat{\vec{k}} | i, \frac{1}{2} \rangle$$

$$= \sum_j \langle i, \frac{1}{2} | \frac{e_T}{4M_T} \left( \frac{e_T}{M_T} - \frac{2e_j}{m_j} \right) \sigma_j^+ i\sqrt{2} (P_z R_z - R_z P_z) | i, -\frac{1}{2} \rangle$$

$$= \sum_j \langle i, \frac{1}{2} | \frac{e_T}{4M_T} \left( \frac{e_T}{M_T} - \frac{2e_j}{m_j} \right) \sigma_j^+ \sqrt{2} | i, -\frac{1}{2} \rangle. \tag{65}$$

Combining Eq. 64 and Eq. 65 gives Eq. 24.

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# Figure captions

1. The $Q^2$ dependence of the longitudinal amplitudes $S^P_{1/2}$ for the resonances $S_{11}(1535)$, $D_{13}(1520)$ and $F_{15}(1680)$.
2. The $Q^2$ dependence of the spin dependent sum rule of $g_1(x, Q^2)$ in the resonance region. The solid and dash lines represent the calculations with and without the quantity $\sigma_{TS}$.
3. The $Q^2$ dependence of the spin dependent sum rule of $g_2(x, Q^2)$ in the resonance region. The solid and dash lines represent the calculations with and without the quantity $\sigma_{TS}$. The dot-dashed line comes from Ref. [21], see text.
4. The estimate of the sum rule $\int_0^1 g_1^p(x, Q^2)dx$. The nonresonant contribution comes from the result in Ref. [5]. The solid and dash lines correspond to the evaluations with and without the quantity $\sigma_{TS}$.

Table 1: Transition matrix elements between the nucleon and baryon resonances in the $SU(6)\otimes O(3)$ symmetry limit. The full matrix elements are obtained by multiplying the entries in this table by a factor $\sqrt{\frac{2\pi}{k_0}}2\mu m_q e^{-\frac{\mathbf{k}^2}{6\alpha^2}}$, and $S^n_{\frac{1}{2}} = S^p_{\frac{1}{2}}$ for $\Delta$ states.

| Multiplet | States | Proton | Neutron |
|---|---|---|---|
| $[70,1^-]_1$ | $N(^2P_M)\frac{1}{2}^{-1}$ | $\frac{1}{3\sqrt{2}}\frac{\lvert\mathbf{k}\rvert}{\alpha}\left(1+\frac{\alpha^2}{6m_q^2}\right)$ | $-\frac{1}{3\sqrt{2}}\frac{\lvert\mathbf{k}\rvert}{\alpha}\left(1+\frac{\alpha^2}{6m_q^2}\right)$ |
| | $N(^2P_M)\frac{3}{2}^{-1}$ | $-\frac{1}{3}\frac{\lvert\mathbf{k}\rvert}{\alpha}\left(1-\frac{\alpha^2}{12m_q^2}\right)$ | $\frac{1}{3}\frac{\lvert\mathbf{k}\rvert}{\alpha}\left(1-\frac{\alpha^2}{12m_q^2}\right)$ |
| | $N(^4P_M)\frac{1}{2}^{-1}$ | $\frac{1}{36\sqrt{2}}\frac{\alpha\lvert\mathbf{k}\rvert}{m_q^2}$ | $-\frac{1}{108\sqrt{2}}\frac{\alpha\lvert\mathbf{k}\rvert}{m_q^2}$ |
| | $N(^4P_M)\frac{3}{2}^{-1}$ | $\frac{1}{9\sqrt{10}}\frac{\alpha\lvert\mathbf{k}\rvert}{m_q^2}$ | $-\frac{5}{27\sqrt{10}}\frac{\alpha\lvert\mathbf{k}\rvert}{m_q^2}$ |
| | $N(^4P_M)\frac{5}{2}^{-1}$ | $\frac{1}{12\sqrt{10}}\frac{\alpha\lvert\mathbf{k}\rvert}{m_q^2}$ | $-\frac{5}{36\sqrt{10}}\frac{\alpha\lvert\mathbf{k}\rvert}{m_q^2}$ |
| | $\Delta(^2P_M)\frac{1}{2}^{-1}$ | $-\frac{1}{3\sqrt{2}}\frac{\lvert\mathbf{k}\rvert}{\alpha}\left(1-\frac{\alpha^2}{6m_q^2}\right)$ | |
| | $\Delta(^2P_M)\frac{3}{2}^{-1}$ | $\frac{1}{3}\frac{\lvert\mathbf{k}\rvert}{\alpha}\left(1+\frac{\alpha^2}{12m_q^2}\right)$ | |
| $[56,0^+]_2$ | $N(^2S_{S'})\frac{1}{2}^+$ | $-\frac{1}{3\sqrt{6}}\frac{\mathbf{k}^2}{\alpha^2}$ | $0$ |
| | $\Delta(^4S_{S'})\frac{3}{2}^+$ | $0$ | |
| $[56,2^+]_2$ | $N(^2D_S)\frac{3}{2}^+$ | $-\frac{1}{3\sqrt{15}}\frac{\mathbf{k}^2}{\alpha^2}\left(1+\frac{\alpha^2}{2m_q^2}\right)$ | $-\frac{\mathbf{k}^2}{12\sqrt{15}m_q^2}$ |
| | $N(^2D_S)\frac{5}{2}^+$ | $-\frac{1}{3\sqrt{10}}\frac{\mathbf{k}^2}{\alpha^2}\left(1-\frac{\alpha^2}{3m_q^2}\right)$ | $\frac{\mathbf{k}^2}{9\sqrt{10}m_q^2}$ |
| | $\Delta(^4D_S)\frac{1}{2}^+$ | $-\frac{5\mathbf{k}^2}{72\sqrt{15}m_q^2}$ | |
| | $\Delta(^4D_S)\frac{3}{2}^+$ | $0$ | |
| | $\Delta(^4D_S)\frac{5}{2}^+$ | $\frac{5\sqrt{5}\mathbf{k}^2}{216\sqrt{7}m_q^2}$ | |
| | $\Delta(^4D_S)\frac{7}{2}^+$ | $\frac{5\mathbf{k}^2}{36\sqrt{105}m_q^2}$ | |
| $[70,0^+]_2$ | $N(^2S_{M'})\frac{1}{2}^+$ | $\frac{1}{18}\frac{\mathbf{k}^2}{\alpha^2}$ | $-\frac{1}{18}\frac{\mathbf{k}^2}{\alpha^2}$ |